Problem 12-8

A particle is moving along a straight line such that its position is defined by $s = (10t^2 + 20)$ mm, where t is in seconds. Determine (a) the displacement of the particle during the time interval from $t = 1$ s to $t = 5$ s, (b) the average velocity of the particle during this time interval, and (c) the acceleration when $t = 1$ s.

Solution

The average velocity from $t = 1$ s to $t = 5$ s is

$$v_{\text{avg}} = \frac{\Delta s}{\Delta t} = \frac{s(5) - s(1)}{(5) - (1)} = \frac{[10(5)^2 + 20] - [10(1)^2 + 20]}{4} = 60 \ \frac{\text{mm}}{\text{s}}.$$

Differentiate the given position function to determine the velocity.

$$\begin{aligned} v &= \frac{ds}{dt} \\ &= \frac{d}{dt}(10t^2 + 20) \\ &= 20t \end{aligned}$$

Integrate the velocity from $t = 1$ s to $t = 5$ s to find the displacement during this interval.

$$\begin{aligned} \Delta s &= \int_1^5 v(t)\, dt \\ &= \int_1^5 (20t)\, dt \\ &= 10t^2 \Big|_1^5 \\ &= 10(5^2 - 1^2) \\ &= 240 \text{ mm} \end{aligned}$$

Differentiate the velocity to determine the acceleration.

$$\begin{aligned} a &= \frac{dv}{dt} \\ &= \frac{d}{dt}(20t) \\ &= 20 \end{aligned}$$

Therefore, the acceleration of the particle at $t = 1$ s is

$$a(1) = 20 \ \frac{\text{mm}}{\text{s}^2}.$$